Static Fields of Curved Generally Laminated Anisotropic Plate Strips

Joseph Padovan*
University of Akron, Akron, Ohio

AND

Bernard Gossett†
Goodyear-France, Amiens, France

Theme

LITERALLY a multitude of investigatons on the static mechanical response of laminated plates have appeared. In general, the basic motivation of these studies has been to obtain a clearer understanding of a) the effects of laminate configuration (staking sequence, etc.) on the governing mechanical fields, i.e., bending inplane coupling, etc., and b) the effects of local and global laminate anisotropy. Compared to the flat plate configuration, less work is available which deals directly with the combined effects of curvature, laminate construction, and material anisotropy. With the exception of a paper by Fortier and Rossettos,[1] most of the investigations treating curvature effects in laminated structures have been restricted to complete shells of revolution. In this context, the present paper will consider the combined effects of curvature, laminate configuration, and material anisotropy on the static mechanical fields of curved generally laminated macroscopically anisotropic plate strips.

Contents

For the present purposes, the curved plate strip is simulated by an arbitrary sector of a generally laminated cylindrical shell with arbitrary edge conditions. The general solution of the governing Donnell type Kirchhoff shell theory[2] is facilitated through the use of complex series expansions.[3,4] Referring to Fig. 1, the position of a point of a cylindrical shell sector is given by x, θ, and z which denote, respectively, the axial, circumferential, and radial coordinates. Furthermore, R is the radius of the sector; $h_k - h_{k-1}$, the kth lamina thickness, and γ, the included angle depicted in Fig. 1. Considering Donnell type Kirchhoff shell theory, the appropriate constitutive equations for a generally laminated cylinder are

$$\begin{bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\ & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} \\ & & A_{33} & B_{13} & B_{23} & B_{33} \\ & & & D_{11} & D_{12} & D_{13} \\ & \text{Symmetric} & & & D_{22} & D_{23} \\ & & & & & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{x\theta} \\ \kappa_x \\ \kappa_\theta \\ \kappa_{x\theta} \end{bmatrix} \tag{1}$$

where $N_x, \ldots, M_x, \ldots$ are inplane and bending resultants and $\varepsilon_x, \ldots, \kappa_x, \ldots$ are inplane strains and rotations, respectively. The stiffness elements A_{ij}, B_{ij}, and D_{ij} appearing in Eq. (1) are defined by[2]

$$\langle A_{lk}, B_{lk}, D_{lk} \rangle = \sum_{s=1}^{S} E_{lk}^{(s)}(\alpha_s) \times \{h_{s-1} - h_s, (h_{s-1}^2 - h_s^2)/2, (h_{s-1}^3 - h_s^3)/3\} \tag{2}$$

such that $E_{lk}^{(s)}(\alpha_s)$ are 3-D stiffness properties for which α_s represents the fiber orientation of the sth lamina as depicted in Fig. 1.

For the present paper, since the individual plies are considered fiber reinforced, the material properties used to develop the A, B, and D partitions of Eq. (1) were obtained in the manner of Ref. 2. Using the nomenclature of Ref. 2, the fiber and matrix properties chosen are given by

$$E_m = 0.5 \times 10^6 \text{ psi}, \quad E_f = 0.6 \times 10^8 \text{ psi}, \quad G_m = 0.185 \times 10^6 \text{ psi}, \quad G_f = 0.25 \times 10^8 \text{ psi},$$
$$V_f = 0.5, \quad U_f = 0.2, \quad U_m = 0.35$$

The boundary conditions considered herein consist of simple-simple supports for which $M_\theta(\theta = 0) = 0$ and $M_\theta(\theta = \gamma) = \sigma \cos \pi x/L$.

Obviously, for fiber orientations that yield macroscopically orthotropic situations, $(A_{13}, A_{23}, \ldots, D_{23} \equiv 0)$, the even nature of the implied boundary conditions denoted previously cause N_θ and M_θ fields which are identically zero for $x = L/2$ and $\theta \in [0, \gamma]$. Hence, the $N_\theta(L/2, \theta)$ and $M_\theta(L/2, \theta)$ fields will be used as a simple reference base for a comparison of the macroscopic anisotropic effects of fiber orientation.

Figures 1 and 2 present various aspects of the effects of fiber orientation on the N_θ field of a four-ply symmetric laminate $(\alpha, -\alpha, -\alpha, \alpha)$. In particular, for $\gamma = 270°$, Figs. 1 and 2 present the effects of variations in α on the said fields. For this case,

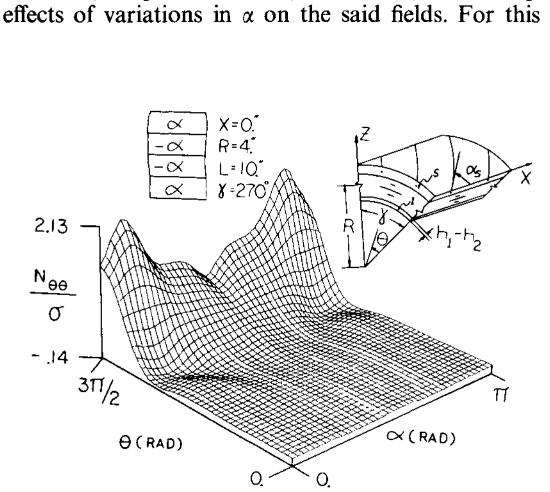

Fig. 1 Effects of α on $N_\theta(0, \theta)$ field of four-ply symmetric laminate for which $\gamma = 270°$.

Received April 16, 1973; synoptic received September 24, 1973; revision received November 27, 1973. Full paper available from National Technical Information Service, Springfield, Va., 22151, as N74-16593 at the standard price (available upon request).

Index categories: Structural Composite Materials (Including Coatings); Structural Static Analysis.

* Assistant Professor of Mechanical Engineering.

† Engineering Department; formerly Goodyear International Fellow at University of Akron.

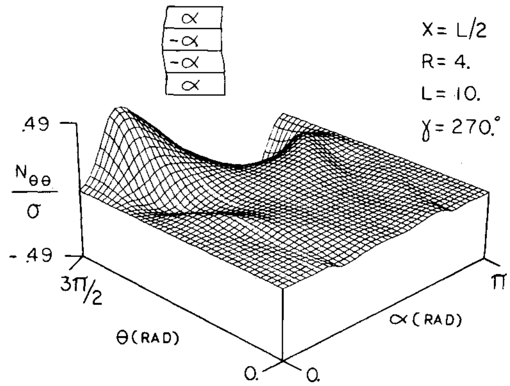

Fig. 2 Effects of α on $N_\theta(L/2, \theta)$ field of four-ply symmetric laminate for which $\gamma = 270°$.

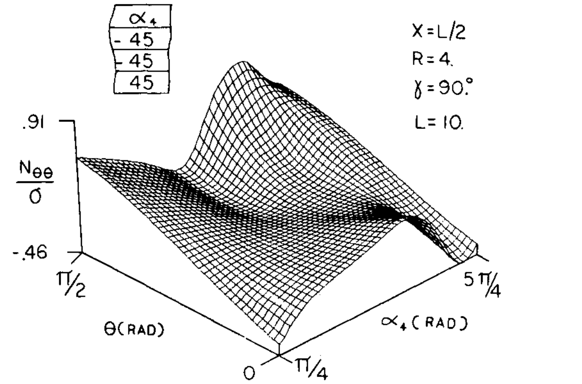

Fig. 4 Effects of fiber misalignments on $N_\theta(L/2, \theta)$ field of symmetrically plied laminate.

although the said fields undergo rapid decay in the θ spatial variable, fairly extensive topological redistributions occur as α is varied in the domain $\alpha \in (0, \pi)$. The nonzero nature of the $N_\theta(L/2, \theta)$ field illustrated in Fig. 2 is directly due to the macroscopic anisotropy caused by fiber orientation. Had the usual "specially" orthotropic assumption been used, the $N_\theta(L/2, \theta)$ field would have been identically zero.

For a balanced four-layered strip (45, −45, −45, 45) of given arc length (γR), the effects of the radius of curvature R on the $M_\theta(L/2, \theta)$ field is depicted in Fig. 3. As is clearly seen in this figure, for increasing R, the effects of fiber orientation on M_θ become more pronounced. This is mainly due to the fact that for $R \gg 0$, the bending and inplane fields uncouple. Hence, the full effects of the stated boundary conditions are supported solely by the bending fields. In the limiting case of $R \to \infty$, the said field attains the flat plate values recently obtained by Padovan.[3]

Figure 4 illustrates various aspects of the effects of possible fiber misalignments in the plies of balanced and alternately plied

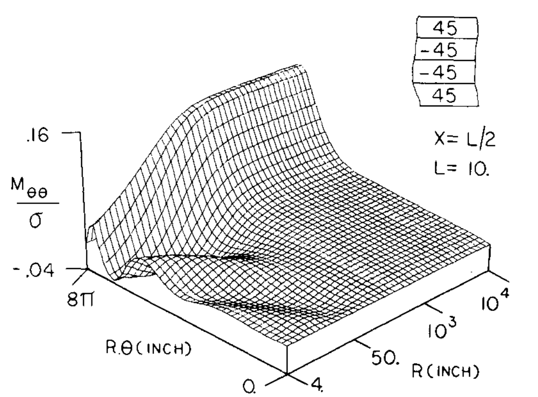

Fig. 3 Effects of R on $M_\theta(L/2, \theta)$ field of four-ply symmetric laminate of given arc length (γR).

laminates. In particular, the effects of fiber misalignments on the $N_\theta(L/2, \theta)$ field of a four-ply symmetrically laminated curved strip (α, −45, −45, 45) is shown. As can be seen, the effect of fiber misalignments causes significant topological redistributions in the said field.

Based on the more extensive numerical results given in the full paper, the following important conclusions were reached.

1) In general, the effects of macroscopic anisotropy may induce significant asymmetries in the displacement, strain, and resultant fields. This effect is independent of the even or odd nature of the edge conditions or surface tractions.

2) The "specially" orthotropic assumption is entirely incapable of revealing the asymmetries noted in (1).

3) For symmetrically plied laminated curved strips, the anisotropic effects of ply orientation can cause significant topological redistributions in the governing fields.

4) For symmetrically plied laminates, the global anisotropic effects of ply orientation on the bending fields increase with increasing R. The reverse is the case for the inplane fields.

5) For alternately plied laminates, in contrast to (4), the global effects of macroscopic anisotropy increase with decreasing radius.

6) For symmetrically as well as alternately laminated curved strips, the effects of fiber misalignments can cause significant topological redistributions in the governing shell fields.

References

[1] Fortier, R. C. and Rossettos, J. N., "On the Vibration of Shear Deformable Curved Anisotropic Composite Plates," *Transactions of ASME, Journal of Applied Mechanics,* Vol. 40, Ser. E, No. 1, March 1973, p. 299.

[2] Ashton, J. E., Halpin, J. C., and Petit, P. H., *Primer on Composite Materials,* Technomic, Stamford, Conn., 1969.

[3] Padovan, J., "Inplane and Bending Fields of Anisotropic Generally Laminated Plate Strips," *Journal of Composite Materials,* Vol. 7, Oct. 1973, pp. 536–542.

[4] Padovan, J., "An Exact Solution for Bending Fields in Anisotropic Balanced Ply Laminated Plate Strips," *Transactions of ASME, Journal of Applied Mechanics,* to be published.